

Other Navigational Formulas

The following formulas - although not part of celestial navigation - are of vital interest because they enable the navigator to calculate course and distance from initial positon A to final position B as well as to calculate the final position B from initial position A, course, and distance.

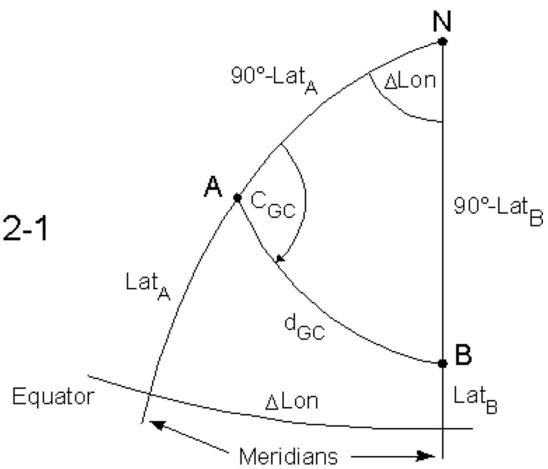
Calculation of Course and Distance

If the coordinates of the initial position A, Lat_A and Lon_A, and the coordinates of the final position B (destination), Lat_B and Lon_B, are known, the navigator has the choice of either traveling along the great circle going through A and B (shortest route) or traveling along the rhumb line going through A and B (slightly longer but easier to navigate).

Great Circle

Great circle distance d_{GC} and course C_{GC} are derived from the navigational triangle (chapter 11) by substituting A for GP, B for AP, d_{GC} for z, and ΔLon (= Lon_B-Lon_A) for LHA (*Fig. 12-1*):

Fig. 12-1



$$d_{GC} = \arccos\left[\sin Lat_A \cdot \sin Lat_B + \cos Lat_A \cdot \cos Lat_B \cdot \cos\left(Lon_B - Lon_A\right)\right]$$

(Northern latitude and eastern longitude are positive, southern latitude and western longitude negative.)

$$C_{GC} = \arccos \frac{\sin Lat_B - \sin Lat_A \cdot \cos d_{GC}}{\cos Lat_A \cdot \sin d_{GC}}$$

C_{GC} has to be converted to the explementary angle, 360°-C_{GC}, if sin (Lon_B-Lon_A) is negative, in order to obtain the true course (0°... 360° clockwise from true north).

C_{GC} is only the initial course and has to be adjusted either continuously or at appropriate intervals because with changing position the angle between the great circle and each local meridian also changes (unless the great circle is the equator or a meridian itself).

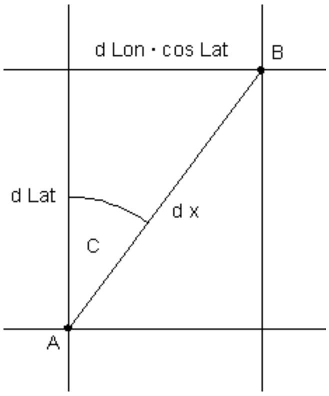
d_{GC} has the dimension of an angle. To convert it to a distance, we multiply d_{GC} by 40031.6/360 (yields the distance in km) or by 60 (yields the distance in nm).

Rhumb Line

A **rhumb line (loxodrome)** is a line on the surface of the earth intersecting all meridians at a constant angle. A vessel steering a constant compass course travels along a rhumb line, provided there is no drift and the magnetic variation remains constant. Rhumb line course C_{RL} and distance d_{RL} are calculated as follows:

First, we imagine traveling the infinitesimally small distance dx from the point of departure, A, to the point of arrival, B. Our course is C (*Fig. 12-2*):

Fig. 12-2



The path of travel, dx, can be considered as composed of a north-south component, dLat, and a west-east component, dLon · cos Lat. The factor cos Lat is the relative circumference of the respective parallel of latitude (equator = 1):

$$\tan C = \frac{d\,Lon \cdot \cos Lat}{d\,Lat}$$

$$\frac{d\,Lat}{\cos Lat} = \frac{1}{\tan C} \cdot d\,Lon$$

If we increase the distance between A (Lat_A, Lon_A) and B (Lat_B, Lon_B), we have to integrate:

$$\int_{Lat\,A}^{Lat\,B} \frac{d\,Lat}{\cos Lat} = \frac{1}{\tan C} \cdot \int_{Lon\,A}^{Lon\,B} d\,Lon$$

$$\ln\left[\tan\left(\frac{Lat_B}{2} + \frac{\pi}{4}\right)\right] - \ln\left[\tan\left(\frac{Lat_A}{2} + \frac{\pi}{4}\right)\right] = \frac{Lon_B - Lon_A}{\tan C}$$

$$\tan C = \frac{Lon_B - Lon_A}{\ln \dfrac{\tan\left(\dfrac{Lat_B}{2} + \dfrac{\pi}{4}\right)}{\tan\left(\dfrac{Lat_A}{2} + \dfrac{\pi}{4}\right)}}$$

Solving for C and measuring angles in degrees, we get:

$$C_{RL} = \arctan \frac{\pi \cdot \left(Lon_B - Lon_A\right)}{180° \cdot \ln \dfrac{\tan\left(\dfrac{Lat_B[°]}{2} + 45°\right)}{\tan\left(\dfrac{Lat_A[°]}{2} + 45°\right)}}$$

$(Lon_B - Lon_A)$ has to be in the range between –180° tand +180°. If it is outside this range, add or subtract 360° before entering the rhumb line course formula.

The arctan function returns values between -90° and +90°. To obtain the true course, $C_{RL,N}$, we apply the following rules:

$$C_{RL,\,N} = \begin{cases} C_{RL} & \text{if} \quad Lat_B > Lat_A \quad \text{AND} \quad Lon_B > Lon_A \\ 180° - C_{RL} & \text{if} \quad Lat_B < Lat_A \quad \text{AND} \quad Lon_B > Lon_A \\ 180° + C_{RL} & \text{if} \quad Lat_B < Lat_A \quad \text{AND} \quad Lon_B < Lon_A \\ 360° - C_{RL} & \text{if} \quad Lat_B > Lat_A \quad \text{AND} \quad Lon_B < Lon_A \end{cases}$$

To find the total length of our path of travel, we calculate the infinitesimal distance dx:

$$d\,x = \frac{d\,Lat}{\cos C}$$

The total length is found through integration:

$$D = \int_0^D d\,x = \frac{1}{\cos C} \cdot \int_{Lat\,A}^{Lat\,B} d\,Lat = \frac{Lat_B - Lat_A}{\cos C}$$

Measuring D in kilometers or nautical miles, we get:

$$D_{RL}[km] = \frac{40031.6}{360} \cdot \frac{Lat_B - Lat_A}{\cos C_{RL}} \qquad D_{RL}[nm] = 60 \cdot \frac{Lat_B - Lat_A}{\cos C_{RL}}$$

If both positions have the same latitude, the distance can not be calculated using the above formulas. In this case, the following formulas apply (C_{RL} is either 90° or 270°.):

$$D_{RL}[km] = \frac{40031.6}{360} \cdot (Lon_B - Lon_A) \cdot \cos Lat \qquad D_{RL}[nm] = 60 \cdot (Lon_B - Lon_A) \cdot \cos Lat$$

Mid latitude

Since the rhumb line course formula is rather complicated, it is mostly replaced by the **mid latitude** formula in everyday navigation. This is an approximation giving good results as long as the distance between both positions is not too large and both positions are far enough from the poles.

Mid latitude course:

$$C_{ML} = \arctan\left(\cos Lat_M \cdot \frac{Lon_B - Lon_A}{Lat_B - Lat_A}\right) \qquad Lat_M = \frac{Lat_A + Lat_B}{2}$$

The true course is obtained by applying the same rules to C_{ML}as to the rhumb line course C_{RL}.

Mid latitude distance:

$$d_{ML}[km] = \frac{40031.6}{360} \cdot \frac{Lat_B - Lat_A}{\cos C_{ML}} \qquad d_{ML}[nm] = 60 \cdot \frac{Lat_B - Lat_A}{\cos C_{ML}}$$

If C_{ML} = 90° or C_{ML} = 270°, apply the following formulas:

$$d_{ML}[km] = \frac{40031.6}{360} \cdot (Lon_B - Lon_A) \cdot \cos Lat \qquad d_{ML}[nm] = 60 \cdot (Lon_B - Lon_A) \cdot \cos Lat$$

Dead Reckoning

Dead reckoning is the navigational term for extrapolating one's new position B (**dead reckoning position, DRP**) from the previous position A, course C, and distance d (calculated from the vessel's average speed and time elapsed). Since dead reckoning can only yield an approximate position (due to the influence of drift, etc.), the mid latitude method provides sufficient accuracy. On land, dead reckoning is more difficult than at sea since it is usually not possible to steer a constant course (apart from driving in large, entirely flat areas like, e.g., salt flats). At sea, the DRP is needed to choose an appropriate (near-by) AP. If celestial observations are not possible and electronic navigation aids are not available, dead reckoning may be the only way of keeping track of one's position.

Calculation of new latitude:

$$Lat_B[\circ] = Lat_A[\circ] + \frac{360}{40031.6} \cdot d[km] \cdot \cos C \qquad Lat_B[\circ] = Lat_A[\circ] + \frac{d[nm]}{60} \cdot \cos C$$

Calculation of new longitude:

$$Lon_B[\circ] = Lon_A[\circ] + \frac{360}{40031.6} \cdot d[km] \cdot \frac{\sin C}{\cos Lat_M} \qquad Lon_B[\circ] = Lon_A[\circ] + \frac{d[nm]}{60} \cdot \frac{\sin C}{\cos Lat_M}$$

If the resulting longitude exceeds +180°, subtract 360°. If it exceeds -180°, add 360°.