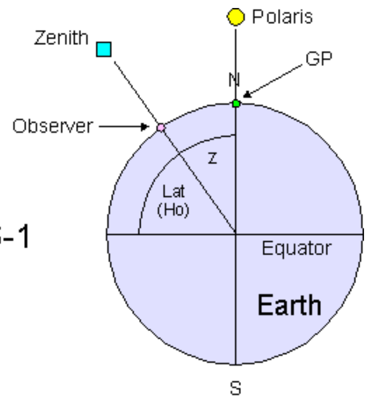


Methods for Latitude and Longitude Measurement

Latitude by Polaris

The observed altitude of a star being vertically above the geographic north pole would be numerically equal to the latitude of the observer (*Fig. 6-1*).

Fig. 6-1



This is nearly the case with the pole star (Polaris). However, since there is a measurable angular distance between Polaris and the polar axis of the earth (presently ca. 1°), the altitude of Polaris is a function of the local hour angle. The altitude of Polaris is also affected by nutation. To obtain the accurate latitude, several corrections have to be applied:

$$Lat = Ho - 1° + a_0 + a_1 + a_2$$

The corrections a_0, a_1, and a_2 depend on LHA_{Aries}, the observer's estimated latitude, and the number of the month. They are given in the Polaris Tables of the Nautical Almanac [12]. To extract the data, the observer has to know his approximate position and the approximate time.

When using a computer almanac instead of the N. A., we can calculate Lat with the following simple procedure. Lat_E is our estimated latitude, Dec is the declination of Polaris, and t is the meridian angle of Polaris (calculated from GHA and our estimated longitude). Hc is the computed altitude, Ho is the observed altitude (see chapter 4).

$$Hc = \arcsin\left(\sin Lat_E \cdot \sin Dec + \cos Lat_E \cdot \cos Dec \cdot \cos t\right)$$

$$\Delta H = Ho - Hc$$

Adding the altitude difference, ΔH, to the estimated latitude, we obtain the improved latitude:

$$Lat \approx Lat_E + \Delta H$$

The error of Lat is smaller than 0.1' when Lat_E is smaller than 70° and when the error of Lat_E is smaller than 2°, provided the exact longitude is known.. In polar regions, the algorithm becomes less accurate. However, the result can be improved by iteration (substituting Lat for Lat_E and repeating the calculation). Latitudes greater than 85° should be avoided because a greater number of iterations might be necassary. The method may lead to erratic results when the observer is close to the north pole ($Lat_E \approx Dec_{Polaris}$). An error in Lat resulting from an error in longitude is **not** decreased by iteration. However, this error is always smaller than 1' when the error in longitude is smaller than 1°.

Noon Latitude (Latitude by Maximum Altitude)

This is a very simple method enabling the observer to determine latitude by measuring the maximum altitude of the sun (or any other object). No accurate time measurement is required. The altitude of the sun passes through a flat maximum **approximately** (see noon longitude) at the moment of upper meridian passage (**local apparent noon, LAN**) when the GP of the sun has the same longitude as the observer and is either north or south of him, depending on the declination of the sun and observer's geographic latitude. The observer's latitude is easily calculated by forming the algebraic sum or difference of the declination and observed zenith distance z (90°-Ho) of the sun, depending on whether the sun is north or south of the observer (*Fig. 6-2*).

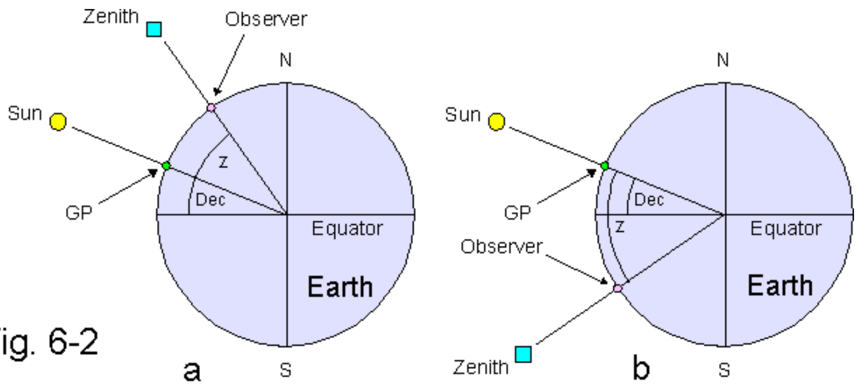


Fig. 6-2

1. Sun south of observer (*Fig. 6-2a*): $$Lat = Dec + z = Dec - Ho + 90°$$

2. Sun north of observer (*Fig. 6-2b*): $$Lat = Dec - z = Dec + Ho - 90°$$

Northern declination is positive, southern negative.

Before starting the observations, we need a rough estimate of our current longitude to know the time (GMT) of meridian transit. We look up the time of Greenwich meridian transit of the sun on the daily page of the Nautical Almanac and add 4 minutes for each degree of western longitude or subtract 4 minutes for each degree of eastern longitude. To determine the maximum altitude, we start observing the sun approximately 15 minutes before meridian transit. We follow the increasing altitude of the sun with the sextant, note the maximum altitude when the sun starts descending again, and apply the usual corrections.

We look up the declination of the sun at the approximate time (GMT) of local meridian passage on the daily page of the Nautical Almanac and apply the appropriate formula.

Historically, noon latitude and latitude by Polaris are among the oldest methods of celestial navigation.

Ex-Meridian Sight

Sometimes, it may be impossible to measure the maximum altitude of the sun. For example, the sun may be obscured by a cloud at this moment. If we have a chance to measure the altitude of the sun a few minutes before or after meridian transit, we are still able to find our exact latitude by reducing the observed altitude to the meridian altitude, provided we know our exact longitude (see below) and have an estimate of our latitude. The method is similar to the one used with the pole star. First, we need the time (UT) of local meridian transit (eastern longitude is positive, western longitude negative):

$$T_{Transit}\left[h\right] = 12 - EoT\left[h\right] - \frac{Lon\left[°\right]}{15}$$

The meridian angle of the sun, t, is calculated from the time of observation (GMT):

$$t\left[°\right] = 15 \cdot \left(T_{Observation}\left[h\right] - T_{Transit}\left[h\right]\right)$$

Starting with our estimated Latitude, Lat_E, we calculate the altitude of the sun at the time of observation. We use the altitude formula from chapter 4:

$$Hc = \arcsin\left(\sin Lat_E \cdot \sin Dec + \cos Lat_E \cdot \cos Dec \cdot \cos t \right)$$

Dec refers to the time of observation. We calculate the difference between observed and calculated altitude:

$$\Delta H = Ho - Hc$$

We calculate an improved latitude, $Lat_{improved}$:

$$Lat_{improved} \approx Lat_E \pm \Delta H$$

(sun north of observer: $+\Delta H$, sun south of observer: $-\Delta H$)

The exact latitude is obtained by iteration, i. e., we substitute $Lat_{improved}$ for Lat_E and repeat the calculations until the obtained latitude is virtually constant. Usually, no more than one or two iterations are necessary. The method has a few limitations and requires critical judgement. The meridian angle should be small, compared with the zenith distance of the sun. Otherwise, a greater number of iterations may be necessary. The method may yield erratic results if Lat_E is similar to Dec. A sight should be discarded when the observer is not sure if the sun is north or south of his position.

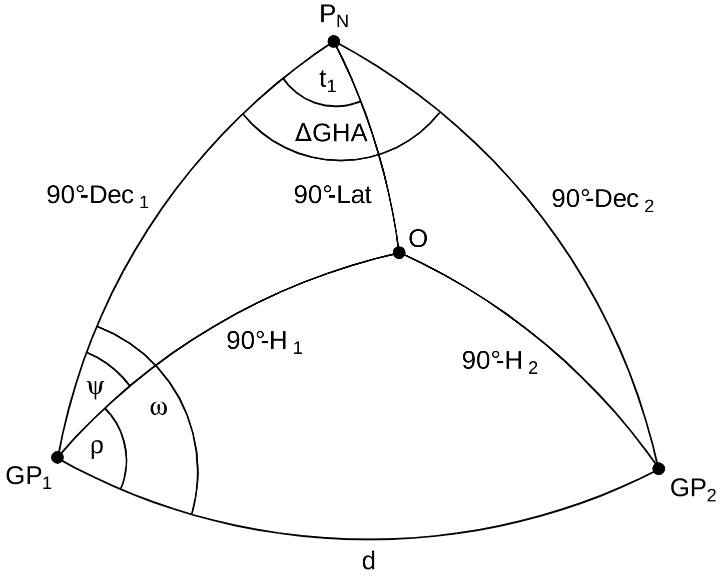
The influence of a longitude error on the latitude thus obtained is **not** decreased by iteration.

Latitude by two altitudes

Even if no estimated longitude is available, the exact latitude can still be found by observation of two celestial bodies. The required quantities are Greenwich hour angle, declination, and observed altitude of each body [7].

The calculations are based upon spherical triangles (see chapter 10 & chapter 11). In *Fig. 6-3*, P_N denotes the north pole, O the observer's unknown position, GP_1 the geographic position of the first body, and GP_2 the position of the second body.

Fig. 6-3



First, we consider the spherical triangle [GP_1, P_N, GP_2]. *Fig. 6-3* shows only one of several possible configurations. O may as well be outside the triangle [GP_1, P_N, GP_2]. We form the difference of both Greenwich hour angles, ΔGHA:

$$\Delta GHA = \left| GHA_2 - GHA_1 \right|$$

Using the law of cosines for sides (chapter 10), we calculate the great circle distance between GP_1 and GP_2, d.

$$\cos d = \sin Dec_1 \cdot \sin Dec_2 + \cos Dec_1 \cdot \cos Dec_2 \cdot \cos(\Delta GHA)$$

$$d = \arccos\left[\sin Dec_1 \cdot \sin Dec_2 + \cos Dec_1 \cdot \cos Dec_2 \cdot \cos(\Delta GHA)\right]$$

Now we solve the same triangle for the angle ω, the horizontal distance between P_N and GP_2, measured at GP_1:

$$\cos \omega = \frac{\sin Dec_2 - \sin Dec_1 \cdot \cos d}{\cos Dec_1 \cdot \sin d}$$

$$\omega = \arccos\left(\frac{\sin Dec_2 - \sin Dec_1 \cdot \cos d}{\cos Dec_1 \cdot \sin d}\right)$$

For the spherical triangle [GP_1, O, GP_2], we calculate the angle ρ, the horizontal distance between O and GP_2, measured at GP_1.

$$\cos \rho = \frac{\sin H_2 - \sin H_1 \cdot \cos d}{\cos H_1 \cdot \sin d}$$

$$\rho = \arccos\left(\frac{\sin H_2 - \sin H_1 \cdot \cos d}{\cos H_1 \cdot \sin d}\right)$$

We calculate the angle ψ, the horizontal distance between P_N and O, measured at GP_1. There are two solutions (ψ_1 and ψ_2) since cos ρ = cos (-ρ):

$$\psi_1 = \left|\omega - \rho\right| \qquad \psi_2 = \omega + \rho$$

The circles of equal altitude intersect each other at two points. The corresponding positions are on opposite sides of the great circle going through GP_1 and GP_2 (not shown in *Fig. 6-3*). Using the law of cosines for sides again, we solve the spherical triangle [GP_1, P_N, O] for Lat. Since we have two solutions for ψ, we obtain two possible latitudes, Lat_1 and Lat_2.

$$\sin Lat_1 = \sin H_1 \cdot \sin Dec_1 + \cos H_1 \cdot \cos Dec_1 \cdot \cos \psi_1$$

$$Lat_1 = \arcsin\left(\sin H_1 \cdot \sin Dec_1 + \cos H_1 \cdot \cos Dec_1 \cdot \cos \psi_1\right)$$

$$\sin Lat_2 = \sin H_1 \cdot \sin Dec_1 + \cos H_1 \cdot \cos Dec_1 \cdot \cos \psi_2$$

$$Lat_2 = \arcsin\left(\sin H_1 \cdot \sin Dec_1 + \cos H_1 \cdot \cos Dec_1 \cdot \cos \psi_2\right)$$

We choose the value nearest to our estimated latitude. The other one is discarded. If both solutions are very similar and a clear distinction is not possible, one of the sights should be discarded, and a body with a more favorable position should be chosen.

Although the method requires more complicated calculations than, e. g., a latitude by Polaris, it has the advantage that measuring two altitudes usually takes less time than finding the maximum altitude of a single body. Moreover, if fixed stars are observed, even a chronometer error of several hours has no significant influence on the resulting latitude since ΔGHA and both declinations change very slowly in this case.

When the horizontal distance between the observed bodies is in the vicinity of 0° or 180°, the observer's position is close to the great circle going through $GP_1$and GP_2. In this case, the two solutions for latitude are similar, and finding which one corresponds with the actual latitude may be difficult (depending on the quality of the estimate). The resulting latitudes are also close to each other when the observed bodies have approximately the same Greenwich hour angle.

Noon Longitude (Longitude by Equal Altitudes)

Since the earth rotates with an angular velocity of 15° per hour with respect to the mean sun, the time of local meridian transit (local apparent noon) of the sun, $T_{Transit}$, can be used to calculate the observer's longitude:

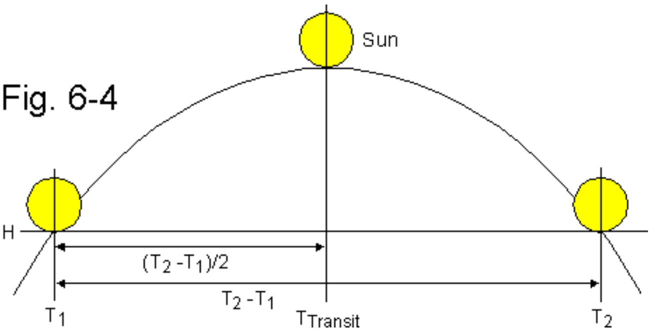
$$Lon[°] = 15 \cdot (12 - T_{Transit}[h] - EoT_{Transit}[h])$$

$T_{Transit}$ is measured as GMT (decimal format). The correction for EoT at the time of meridian transit, $EoT_{Transit}$, has to be made because the apparent sun, not the mean sun, is observed (see chapter 3). Since the Nautical Almanac contains only values for EoT (see chapter 3) at 0:00 GMT and 12:00 GMT of each day, $EoT_{Transit}$ has to be found by interpolation.

Since the altitude of the sun - like the altitude of any celestial body - passes through a rather flat maximum, the time of peak altitude is difficult to measure. The exact time of meridian transit can be derived, however, from the times of two equal altitudes of the sun.

Assuming that the sun moves along a symmetrical arc in the sky, $T_{Transit}$ is the mean of the times corresponding with a chosen pair of equal altitudes of the sun, one occurring before LAN (T_1), the other past LAN (T_2) (*Fig. 6-4*):

Fig. 6-4



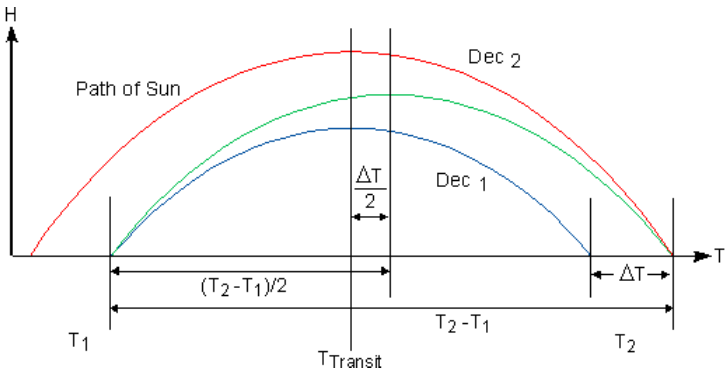
$$T_{Transit} = \frac{T_1 + T_2}{2}$$

In practice, the times of equal altitudes of the sun are measured as follows:

In the morning, the observer records the time T_1 corresponding with a chosen altitude, H. In the afternoon, the time T_2 is recorded when the descending sun passes through the same altitude again. Since only times of equal altitudes are measured, no altitude correction is required. The interval T_2-T_1 should be greater than approx. 2 hours.

Unfortunately, the arc of the sun is only symmetrical with respect to $T_{Transit}$ if the sun's declination is constant during the observation interval. This is approximately the case around the times of the solstices. During the rest of the year, particularly at the times of the equinoxes, $T_{Transit}$ differs significantly from the mean of T_1 and T_2 due to the changing declination of the sun. *Fig. 6-5* shows the altitude of the sun as a function of time and illustrates how the changing declination affects the apparent path of the sun in the sky, resulting in a time difference, ΔT.

Fig. 6-5



The blue line shows the path of the sun for a given, constant declination, Dec_1. The red line shows how the path would look with a different declination, Dec_2. In both cases, the apparent path of the sun is symmetrical with respect to $T_{Transit}$. However, if the sun's declination varies from Dec_1 at T_1 to Dec_2 at T_2, the path shown by the green line will result.

Now, T_1 and T_2 are no longer symmetrical to $T_{Transit}$. The sun's meridian transit occurs before $(T_1+T_2)/2$ if the sun's declination changes toward the observer's parallel of latitude, like shown in *Fig. 6-5*. Otherwise, the meridian transit occurs after $(T_1+T_2)/2$. Since time and local hour angle (or meridian angle) are proportional to each other, a systematic error in longitude results.

The error in longitude is negligible around the times of the solstices when Dec is almost constant, and is greatest (up to several arcminutes) at the times of the equinoxes when the rate of change of Dec is greatest (approx. 1'/h). Moreover, the error in longitude increases with the observer's latitude and may be quite dramatic in polar regions.

The obtained longitude can be improved, if necessary, by application of the **equation of equal altitudes** [5]:

$$\Delta t \approx \left(\frac{\tan Lat}{\sin t_2} - \frac{\tan Dec_2}{\tan t_2} \right) \cdot \Delta Dec \qquad\qquad \Delta Dec = Dec_2 - Dec_1$$

Δt is the change in the meridian angle, t, which cancels the change in altitude resulting from a small change in declination, ΔDec. Lat is the observer's latitude. If the accurate latitude is not known, an estimated latitude may be used. t_2 is the meridian angle of the sun at T_2. Since we do not know the exact value for t_2 initially, we start our calculations with an approximate value calculated from T_1 and T_2:

$$t_2[°] \approx \frac{15 \cdot (T_2[h] - T_1[h])}{2}$$

We denote the improved value for T_2 by T_2*.

$$T_2^*[h] = T_2[h] - \Delta T[h] = T_2[h] - \frac{\Delta t[°]}{15}$$

At T_2*, the sun would pass through the same altitude as measured at T_1 if Dec did not change during the interval of observation. Accordingly, the improved time of meridian transit is:

$$T_{Transit} = \frac{T_1 + T_2^*}{2}$$

The residual error resulting from the initial error of t_2 is usually not significant. It can be decreased, if necessary, by iteration. Substituting T_2* for T_2, we get the improved meridian angle, t_2*:

$$t_2^*[°] \approx \frac{15 \cdot \left(T_2^*[h] - T_1[h]\right)}{2}$$

With the improved meridian angle t_2*, we calculate the improved correction Δt*:

$$\Delta t^* \approx \left(\frac{\tan Lat}{\sin t_2^*} - \frac{\tan Dec_2}{\tan t_2^*} \right) \cdot \Delta Dec$$

Finally, we obtain a more accurate time value, T_2**:

$$T_2^{**}[h] = T_2[h] - \frac{\Delta t^*[°]}{15}$$

And, accordingly:

$$T_{Transit} = \frac{T_1 + T_2^{**}}{2}$$

The error of ΔDec should be as small as possible. Calculating ΔDec with a high-precision computer almanac is preferable to extracting it from the Nautical Almanac. When using the Nautical Almanac, ΔDec should be calculated from the daily change of declination to keep the rounding error as small as possible.

Although the equation of equal altitudes is strictly valid only for an infinitesimal change of Dec, dDec, it can be used for a measurable change, ΔDec, (up to several arcminutes) as well without sacrificing much accuracy. Accurate time measurement provided, the residual error in longitude rarely exceeds ±0.1'.

Theory of the equation of equal altitudes

The equation of equal altitudes is derived from the altitude formula (see chapter 4) using **differential calculus**:

$$\sin H \;=\; \sin Lat \cdot \sin Dec + \cos Lat \cdot \cos Dec \cdot \cos t$$

First, we need to know how a small change in declination would affect sin H. We form the partial derivative with respect to Dec:

$$\frac{\partial\left(\sin H\right)}{\partial\, Dec} \;=\; \sin Lat \cdot \cos Dec - \cos Lat \cdot \sin Dec \cdot \cos t$$

Thus, the change in sin H caused by an infinitesimal change in declination, d Dec, is:

$$\frac{\partial\left(\sin H\right)}{\partial\, Dec} \cdot d\,Dec \;=\; \left(\sin Lat \cdot \cos Dec - \cos Lat \cdot \sin Dec \cdot \cos t\right) \cdot d\,Dec$$

Now, we form the partial derivative with respect to t in order to find out how a small change in the meridian angle would affect sin H:

$$\frac{\partial\left(\sin H\right)}{\partial t} \;=\; -\cos Lat \cdot \cos Dec \cdot \sin t$$

The change in sin H caused by an infinitesimal change in the meridian angle, dt, is:

$$\frac{\partial\left(\sin H\right)}{\partial t} \cdot d\,t \;=\; -\cos Lat \cdot \cos Dec \cdot \sin t \cdot d\,t$$

Since we want both effects to cancel each other, the total differential has to be zero:

$$\frac{\partial\left(\sin H\right)}{\partial\, Dec} \cdot d\;Dec + \frac{\partial\left(\sin H\right)}{\partial t} \cdot d\,t \;=\; 0$$

$$-\frac{\partial\left(\sin H\right)}{\partial t} \cdot d\,t \quad = \quad \frac{\partial\left(\sin H\right)}{\partial\, Dec} \cdot d\;Dec$$

$$\cos Lat \cdot \cos Dec \cdot \sin t \cdot d\,t \;=\; \left(\sin Lat \cdot \cos Dec - \cos Lat \cdot \sin Dec \cdot \cos t\right) \cdot d\,Dec$$

$$d\,t \;=\; \frac{\sin Lat \cdot \cos Dec - \cos Lat \cdot \sin Dec \cdot \cos t}{\cos Lat \cdot \cos Dec \cdot \sin t} \cdot d\,Dec$$

$$d\,t \;=\; \left(\frac{\tan Lat}{\sin t} - \frac{\tan Dec}{\tan t}\right) \cdot d\,Dec$$

$$\Delta t \;\approx\; \left(\frac{\tan Lat}{\sin t} - \frac{\tan Dec}{\tan t}\right) \cdot \Delta\,Dec$$

Longitude Measurement on a Traveling Vessel

On a traveling vessel, we have to take into account not only the influence of varying declination but also the effects of changing latitude and longitude on the altitude of the body during the observation interval. Differentiating sin H (altitude formula) with respect to Lat, we get:

$$\frac{\partial\left(\sin H\right)}{\partial\,Lat} \;=\; \cos\,Lat \cdot \sin Dec - \sin\,Lat \cdot \cos Dec \cdot \cos\,t$$

Again, the total differential is zero because the combined effects of latitude and meridian angle cancel each other with respect to their influence on sin H:

$$\frac{\partial\left(\sin H\right)}{\partial\,Lat} \cdot d\,Lat + \frac{\partial\left(\sin H\right)}{\partial\,Dec} \cdot d\,t \;=\; 0$$

In analogy with a change in declination, we obtain the following formula for a small change in latitude:

$$d\,t \;=\; \left(\frac{\tan Dec}{\sin t} - \frac{\tan Lat}{\tan t}\right) \cdot d\,Lat$$

The correction for the combined variations in Dec, Lat, and Lon is:

$$\Delta t \;\approx\; \left(\frac{\tan Lat_2}{\sin t_2} - \frac{\tan Dec_2}{\tan t_2}\right) \cdot \Delta Dec \;+\; \left(\frac{\tan Dec_2}{\sin t_2} - \frac{\tan Lat_2}{\tan t_2}\right) \cdot \Delta\,Lat \;-\; \Delta\,Lon$$

ΔLat and ΔLon are the small changes in latitude and longitude corresponding with the path of the vessel traveled between T_1 and T_2. The meridian angle, t_2, has to include a correction for ΔLon:

$$t_2\left[\circ\right] \;\approx\; \frac{15 \cdot \left(T_2\left[h\right] - T_1\left[h\right]\right) - \Delta\,Lon\left[\circ\right]}{2}$$

ΔLat and ΔLon are calculated from the course, C, the velocity over ground, v, and the time elapsed.

$$\Delta\,Lat\left[\,'\,\right] = v\left[\,kn\,\right] \cdot \cos C \cdot \left(T_2\left[\,h\,\right] - T_1\left[\,h\,\right]\right)$$

$$Lat_2 = Lat_1 + \Delta Lat$$

$$\Delta Lon['] = v[kn] \cdot \frac{\sin C}{\cos Lat} \cdot (T_2[h] - T_1[h])$$

$$Lon_2 = Lon_1 + \Delta Lon$$

$$1\, kn(knot) = 1\, nm/h$$

C is measured clockwise from true north (0°...360°). Again, the corrected time of equal altitude is:

$$T_2^*[h] = T_2[h] - \frac{\Delta t[°]}{15}$$

$$T_{Transit} = \frac{T_1 + T_2^*}{2}$$

The longitude calculated from $T_{Transit}$ refers to the observer's position at T_1. The longitude at T_2 is Lon+ΔLon.

The longitude error caused by a change in latitude can be dramatic and requires the navigator's particular attention, even if the vessel travels at a moderate speed. The above considerations clearly demonstrate that determining one's **exact** longitude by equal altitudes of the sun is not as simple as it seems to be at first glance, particularly on a traveling vessel. It is therefore quite natural that with the development of position line navigation (including simple graphic solutions for a traveling vessel), longitude by equal altitudes became less important.

The Meridian Angle of the Sun at Maximum Altitude

Fig. 6-5 shows that the maximum altitude of the sun is slightly different from the altitude at the moment of meridian passage if the declination changes. At maximum altitude, the rate of change of altitude caused by the changing declination cancels the rate of change of altitude caused by the changing meridian angle.
The equation of equal altitude enables us to calculate the meridian angle of the sun at this moment. We divide each side of the equation by the infinitesimal time interval dT:

$$\frac{d\,t}{d\,T} = \left(\frac{\tan Lat}{\sin t} - \frac{\tan Dec}{\tan t} \right) \cdot \frac{d\,Dec}{d\,T}$$

Measuring the rate of change of t and Dec in arcminutes per hour we get:

$$900\,'/h = \left(\frac{\tan Lat}{\sin t} - \frac{\tan Dec}{\tan t} \right) \cdot \frac{d\,Dec[']}{d\,T[h]}$$

Since t is a very small angle, we can substitute tan t for sin t:

$$900 \approx \frac{\tan Lat - \tan Dec}{\tan t} \cdot \frac{d\,Dec[']}{d\,T[h]}$$

Now, we can solve the equation for tan t:

$$\tan t \approx \frac{\tan Lat - \tan Dec}{900} \cdot \frac{d\,Dec[']}{d\,T[h]}$$

Since a small angle (in radians) is nearly equal to its tangent, we get:

$$t[°]\cdot\frac{\pi}{180} \approx \frac{\tan Lat - \tan Dec}{900}\cdot\frac{d\,Dec[']}{d\,T[h]}$$

Measuring t in arcminutes, the equation is stated as:

$$t['] \approx 3.82\cdot(\tan Lat - \tan Dec)\cdot\frac{d\,Dec[']}{d\,T[h]}$$

dDec/dT is the rate of change of declination measured in arcminutes per hour.

The maximum altitude occurs after meridian transit if t is positive, and before meridian transit if t is negative.

For example, at the time of the spring equinox (Dec ≈ 0, dDec/dT ≈ +1'/h) an observer being at +80° (N) latitude would observe the maximum altitude of the sun at t ≈ +21.7', i. e., 86.8 seconds after meridian transit (LAN). An observer at +45° latitude, however, would observe the maximum altitude at t ≈ +3.82', i. e., only 15.3 seconds after meridian transit.

The Maximum Altitude of the Sun

We can use the last equation to evaluate the systematic error of a noon latitude. The latter is based upon the maximum altitude of the sun, not on the altitude at the moment of meridian transit. Following the above example, the observer at 80° latitude would observe the maximum altitude 86.7 seconds after meridian transit.

During this interval, the declination of the sun would have changed from 0 to +1.445" (assuming that Dec is 0 at the time of meridian transit). Using the altitude formula (chapter 4), we get:

$$Hc = \arcsin\left(\sin 80°\cdot\sin 1.445'' + \cos 80°\cdot\cos 1.445''\cdot\cos 21.7'\right) = 10°\,0'\,0.72''$$

In contrast, the calculated altitude at meridian transit would be exactly 10°. Thus, the error of the noon latitude would be -0.72".

In the same way, we can calculate the maximum altitude of the sun observed at 45° latitude:

$$Hc = \arcsin\left(\sin 45°\cdot\sin 0.255'' + \cos 45°\cdot\cos 0.255''\cdot\cos 3.82'\right) = 45°\,0'\,0.13''$$

In this case, the error of the noon latitude would be only -0.13".

The above examples show that even at the times of the equinoxes, the systematic error of a noon latitude caused by the changing declination of the sun is not significant because it is much smaller than other observational errors, e. g., the errors in dip or refraction. A measurable error in latitude can only occur if the observer is very close to one of the poles (tan Lat!). Around the times of the solstices, the error in latitude is practically non-existent.

Time Sight

The process of deriving the longitude from a single altitude of a body (as well as the observation made for this purpose) is called **time sight**. However, this method requires knowledge of the exact latitude, e. g., a noon latitude. Solving the navigational triangle (chapter 11) for the meridian angle, t, we get:

$$t = \pm\arccos\frac{\sin Ho - \sin Lat\cdot\sin Dec}{\cos Lat\cdot\cos Dec}$$

The equation has two solutions, +t and –t, since cos t = cos (–t). Geometrically, this corresponds with the fact that the circle of equal altitude intersects the parallel of latitude at two points.

Using the following formulas and rules, we obtain the longitudes of these points of intersection, Lon_1 and Lon_2:

$$Lon_1 = t - GHA$$

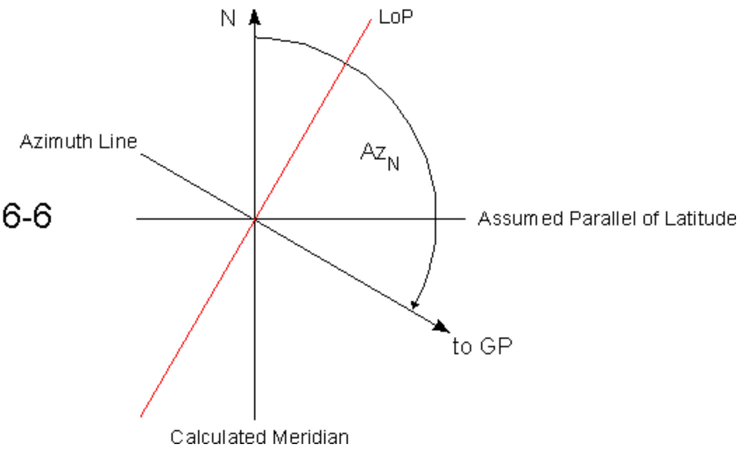
$$Lon_2 = 360° - t - GHA$$

$$\begin{array}{lll} \text{If } Lon_1 < -180° & \rightarrow & Lon_1 + 360° \\ \text{If } Lon_2 < -180° & \rightarrow & Lon_2 + 360° \\ \text{If } Lon_2 > +180° & \rightarrow & Lon_2 - 360° \end{array}$$

A time sight can be used to derive a line of position from a single assumed latitude. After solving the time sight, we plot the assumed parallel of latitude and the calculated meridian.

Next, we calculate the azimuth of the body with respect to the position thus obtained (azimuth formulas, chapter 4) and plot the azimuth line. Our line of position is the perpendicular of the azimuth line going through the calculated position (*Fig. 6-6*).

Fig. 6-6



The latter method is of historical interest only. The modern navigator will certainly prefer the intercept method (chapter 4) which can be used without any restrictions regarding meridian angle (local hour angle), latitude, and declination (see below).

A time sight is not reliable when the body is close to the meridian. Using differential calculus, we can demonstrate that the error of the meridian angle, dt, resulting from an altitude error, dH, varies in proportion with 1/sin t:

$$dt = -\frac{\cos Ho}{\cos Lat \cdot \cos Dec \cdot \sin t} \cdot dH$$

Moreover, dt varies inversely with cos Lat and cos Dec. Therefore, high latitudes and declinations should be avoided as well. The same restrictions apply to *Sumner's* method.

Direct Computation of Position

Combining a **time sight** with a **latitude by two altitudes**, we can find our position, provided we know the exact time. After obtaining our latitude, Lat, from two altitudes (see above), we apply the time sight formula and calculate the meridian angle of the first body, t_1, from the quantities Lat, Dec_1, and H_1 (see *Fig. 6-3*). From t_1, we obtain two possible longitudes. We choose the one nearest to our estimated longitude. This is a rigorous method, not an approximation. It is rarely used, however, since it is more cumbersome than the graphic solutions described in chapter 4.