

Navigational Errors

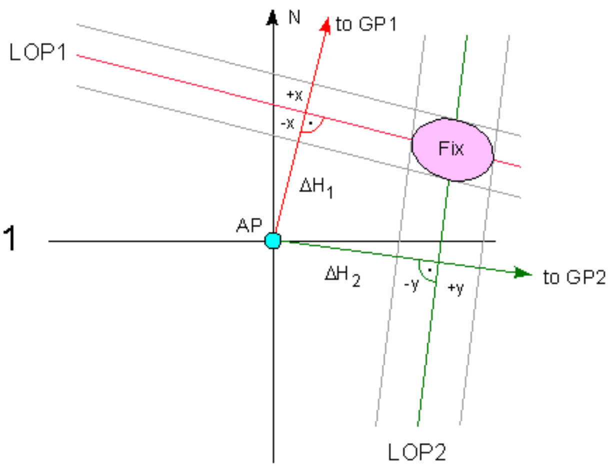
Altitude errors

Apart from systematic errors which can be corrected to a large extent (see chapter 2), observed altitudes always contain random errors caused by ,e.g., heavy seas, abnormal atmospheric refraction, and limited optical resolution of the human eye. Although a good sextant has a mechanical accuracy of ca. 0.1'- 0.3', the **standard deviation** of an altitude measured with a marine sextant is approximately 1' under fair working conditions. The standard deviation may increase to several arcminutes due to disturbing factors or if a bubble sextant or a plastic sextant is used. Altitudes measured with a theodolite are considerably more accurate (0.1'- 0.2').

Due to the influence of random errors, lines of position become indistinct and are better considered as **bands of position**.

Two intersecting bands of position define an **area of position** (ellipse of uncertainty). *Fig. 16-1* illustrates the approximate size and shape of the ellipse of uncertainty for a given pair of LoP's. The standard deviations (±x for the first altitude, ±y for the second altitude) are indicated by grey lines.

Fig. 16-1



The area of position is smallest if the angle between the bands is 90°. The most probable position is at the center of the area, provided the error distribution is symmetrical. Since position lines are perpendicular to their corresponding azimuth lines, objects should be chosen whose azimuths differ by approx. 90° for best accuracy. An angle between 30° and 150°, however, is tolerable in most cases.

When observing more than two bodies, the azimuths should have a roughly symmetrical distribution (**bearing spread**). With multiple observations, the optimum horizontal angle between two adjacent bodies is obtained by dividing 360° by the number of observed bodies (3 bodies: 120°, 4 bodies: 90°, 5 bodies: 72°, 6 bodies: 60°, etc.).

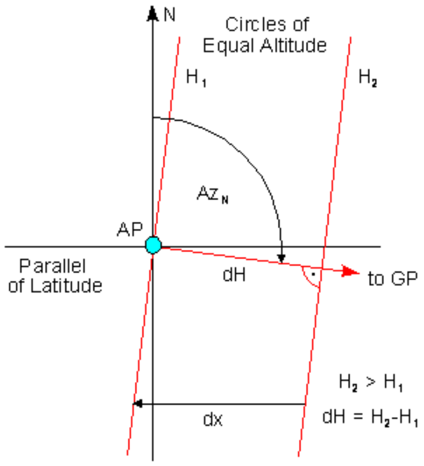
A symmetrical bearing spread not only improves geometry but also compensates for systematic errors like, e.g., index error.

Moreover, there is an optimum range of altitudes the navigator should choose to obtain reliable results. Low altitudes increase the influence of abnormal refraction (random error), whereas high altitudes, corresponding to circles of equal altitude with small diameters, increase geometric errors due to the curvature of LoP's. The generally recommended range to be used is 20° - 70°, but exceptions are possible.

Time errors

The time error is as important as the altitude error since the navigator usually presets the instrument to a chosen altitude and records the time when the image of the body coincides with the reference line visible in the telescope. The accuracy of time measurement is usually in the range between a fraction of a second and several seconds, depending on the rate of change of altitude and other factors. Time error and altitude error are closely interrelated and can be converted to each other, as shown below (*Fig. 16-2*):

Fig. 16-2



The GP of any celestial body travels westward with an angular velocity of approx. 0.25' per second. This is the rate of change of the LHA of the observed body caused by the earth's rotation. The same applies to each circle of equal altitude surrounding GP (tangents shown in *Fig. 6-2*). The distance between two concentric circles of equal altitude (with the altitudes H_1 and H_2) passing through AP in the time interval dt, measured along the parallel of latitude going through AP is:

$$dx\left[nm\right] = 0.25 \cdot \cos Lat_{AP} \cdot dt\left[s\right]$$

dx is also the east-west displacement of a LoP caused by the time error dt. The letter d indicates a small (infinitesimal) change of a quantity (see mathematical literature). cos Lat_{AP} is the ratio of the circumference of the parallel of latitude going through AP to the circumference of the equator (Lat = 0).

The corresponding difference in altitude (the radial distance between both circles of equal altitude) is:

$$dH\left['\right] = \sin Az_N \cdot dx\left[nm\right]$$

Thus, the rate of change of altitude is:

$$\frac{dH\left['\right]}{dt\left[s\right]} = 0.25 \cdot \sin Az_N \cdot \cos Lat_{AP}$$

dH/dt is greatest when the observer is on the equator and decreases to zero as the observer approaches one of the poles. Further, dH/dt is greatest if GP is exactly east of AP (dH/dt positive) or exactly west of AP (dH/dt negative). dH/dt is zero if the azimuth is 0° or 180°. This corresponds to the fact that the altitude of the observed body passes through a minimum or maximum at the instant of meridian transit (dH/dt = 0).

The maximum or minimum of altitude occurs exactly at meridian transit only if the declination of a body is constant. Otherwise, the highest or lowest altitude is observed shortly before or after meridian transit (see chapter 6). The phenomenon is particularly obvious when observing the moon whose declination changes rapidly.

A **chronometer error** is a systematic time error. It influences each line of position in such a way that only the longitude of a fix is affected whereas the latitude remains unchanged, provided the declination does not change significantly (moon!). A chronometer being 1 s fast, for example, displaces a fix by 0.25' to the west, a chronometer being 1 s slow displaces the fix by the same amount to the east. If we know our position, we can calculate the chronometer error from the difference between our true longitude and the longitude found by our observations. If we do not know our longitude, the approximate chronometer error can be found by lunar observations (chapter 7).

Ambiguity

Poor geometry may not only decrease accuracy but may even result in an entirely wrong fix. As the observed horizontal angle (difference in azimuth) between two objects approaches 180°, the distance between the points of intersection of the corresponding circles of equal altitude becomes very small (at exactly 180°, both circles are tangent to each other). Circles of equal altitude with small diameters resulting from high altitudes also contribute to a short distance. A small distance between both points of intersection, however, increases the risk of ambiguity (*Fig. 16-3*).

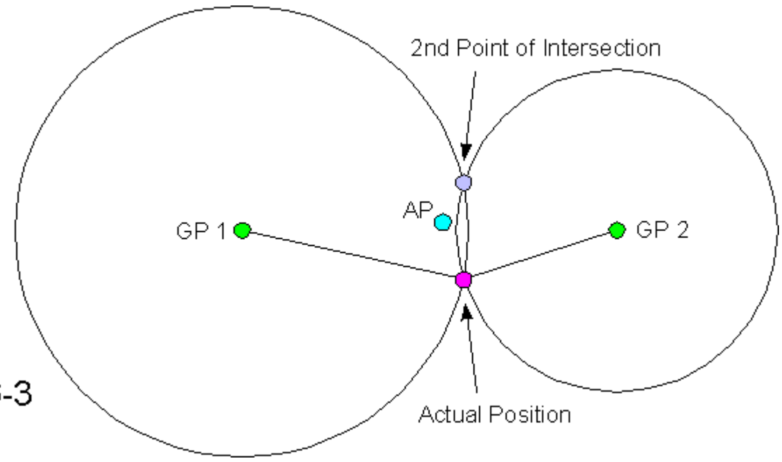


Fig. 16-3

In cases where – due to a horizontal angle near 180° and/or very high altitudes – the distance between both points of intersection is too small, we can not be sure that the assumed position is always close enough to the actual position.

If AP is close to the actual position, the fix obtained by plotting the LoP's (tangents) will be almost identical with the actual position. The accuracy of the fix decreases as the distance of AP from the actual position becomes greater. The distance between fix and actual position increases dramatically as AP approaches the line going through GP1 and GP2 (draw the azimuth lines and tangents mentally). In the worst case, a position error of several hundred or even thousand nm may result !

If AP is exactly on the line going through GP1 and GP2, i.e., equidistant from the actual position and the second point of intersection, the horizontal angle between GP1 and GP2, as viewed from AP, will be 180°. In this case, both LoP's are parallel to each other, and no fix can be found.

As AP approaches the second point of intersection, a fix more or less close to the latter is obtained. Since the actual position and the second point of intersection are symmetrical with respect to the line going through GP1 and GP2, the intercept method can not detect which of both theoretically possible positions is the right one.

Iterative application of the intercept method can only improve the fix if the initial AP is closer to the actual position than to the second point of intersection. Otherwise, an "improved" wrong position will be obtained.

Each navigational scenario should be evaluated critically before deciding if a fix is reliable or not. The distance from AP to the observer's actual position has to be considerably smaller than the distance between actual position and second point of intersection. This is usually the case if the above recommendations regarding altitude, horizontal angle, and distance between AP and actual position are observed.

A simple method to improve the reliability of a fix

Each altitude measured with a sextant, theodolite, or any other device contains systematic and random errors which influence the final result (fix). Systematic errors are more or less eliminated by careful calibration of the instrument used. The influence of random errors decreases if the number of observations is sufficiently large, provided **the error distribution** is symmetrical. Under practical conditions, the number of observations is limited, and the error distribution is more or less unsymmetrical, particularly if an **outlier**, a measurement with an abnormally large error, is present. Therefore, the average result may differ significantly from the true value. When plotting more than two lines of position, the experienced navigator may be able to identify outliers by the shape of the error polygon and remove the associated LoP's. However, the method of least squares, producing an average value, does not recognize outliers and may yield an inaccurate result.

The following simple method takes advantage of the fact that the **median** of a number of measurements is much less influenced by outliers than the **mean** value:

1. We choose a celestial body and measure a series of altitudes. We calculate azimuth and intercept for each observation of said body. The number of measurements in the series has to be odd (3, 5, 7...). The reliability of the method increases with the number of observations.

2. We sort the calculated **intercepts** by magnitude and choose the **median** (the central value in the array of intercepts thus obtained) and its associated azimuth. We discard all other observations of the series.

3. We repeat the above procedure with at least one additional body (or with the same body after its azimuth has become sufficiently different).

4. We plot the lines of position using the azimuth and intercept selected from each series, or use the selected data to calculate the fix with the method of least squares (chapter 4).

The method has been checked with excellent results on land. At sea, where the observer's position usually changes continually, the method has to be modified by advancing AP according to the path of travel between the observations of each series.